# More integration Practice:

- Find the volume of the solid bounded above by $f(x,y) = 2xy + y^2$ below by the x-y plane ~~enclosed~~ over the region enclosed by $y = x$, $y = x^2$, $x = 1$, $x = 4$

- Do the same as above, but now ~~enclosed by~~ over the region bounded by

  $y = x$, $y = x^2$, $x = 0$, $x = 1$

- Find the volume below $z = x^2 y$ above region $1 \leq x \leq 2$ and $3 \leq y \leq 4$ using

$$\int_3^4 \int_1^2 x^2 y \, dx \, dy \qquad \text{and} \qquad \int_1^2 \int_3^4 x^2 y \, dy \, dx$$

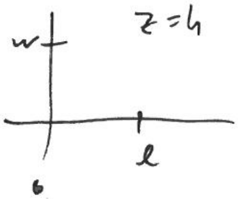
match match

match

Use double integrals to derive the equation for the volume of a rectangular prism

height $= h$
length $= l$
width $= w$

Hint, use $z = h$ over a region with length $l$ and width $w$




$$\int_0^w \int_0^l h \, dx \, dy$$

$$= \int_0^w hx \Big]_0^l dy = \int_0^w hl \, dy = hly \Big]_0^w = hlw$$

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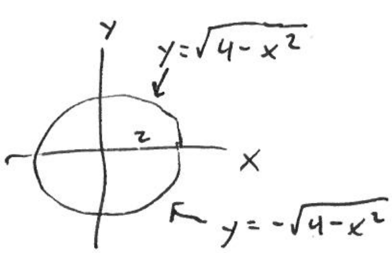
Do Not Solve!
Set up the double integral that you would use to calculate the volume of the ~~hemisphere~~ top half of the sphere given by $4 = x^2 + y^2 + z^2$

$z = 0$




$x^2 + y^2 = 4$

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{4 - x^2 - y^2} \, dy \, dx$$

# 11.1 Taylor Polynomials

The beginning of approximation.

A Polynomial of one variable of degree $n$ is a function of the form

$$p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

$a_0, a_1, \dots, a_n$ are constants and $a_n \neq 0$

Our goal: Use polynomials to approximate more complicated functions.

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The $n^{th}$ Taylor polynomial of a function $f(x)$ near point $x = a$ is

$$\boxed{P_n(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n}$$

Recall

$n!$ is "n-factorial"

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$$\frac{7!}{5!} = \frac{7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 7 \cdot 6$$

$f^{(n)}(x)$ is the $n^{th}$ derivative of $f$

Not an exponent.

$$f'''''''(x) = f^{(7)}(x)$$

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Ex Find the $5^{th}$ degree Taylor polynomial of

$f(x) = \sin(x)$ @ $x = 0$

- So need 5 derivatives

$f(x) = \sin(x)$ $\quad$ $f(0) = 0$

$f'(x) = \cos(x)$ $\quad$ $f'(0) = 1$

$f''(x) = -\sin(x)$ $\quad$ $f''(0) = 0$

$f'''(x) = -\cos(x)$ $\quad$ $f'''(0) = -1$

$f^{(4)}(x) = \sin(x)$ $\quad$ $f^{(4)}(0) = 0$

$f^{(5)}(x) = \cos(x)$ $\quad$ $f^{(5)}(0) = 1$

$$P_5(x) = 0 + \frac{1}{1!}(x-0) + \frac{0}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3 + \frac{0}{4!}(x-0)^4 + \frac{1}{5!}(x-0)^5$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

do you see the pattern?

next comes $\frac{-x^7}{7!}$, $\frac{x^9}{9!}$